Mathematical Models in Epidemiology
Fourth CI^{2}MA Workshop, Universidad de Concepción, June 18 and 19, 2018
Auditorio Alamiro Robledo, Facultad de Ciencias Físicas y Matemáticas
Organizers[1]: Raimund Bürger & Luis Miguel Villada

Programme

Monday, June 18, 2018

15.00 **Gerardo Chowell** (School of Public Health, Georgia State University, Atlanta, GA, USA):
The scaling of epidemic growth in the spread of infectious diseases

15.40 **Tetsuro Kobayashi** (School of Public Health, Georgia State University, Atlanta, GA, USA):
Spatio-temporal and socio-demographic patterns of Chikungunya, Dengue, and Zika infections in Mexico in 2016

16.20 **Coffee break**

16.40 **Katia Vogt** (Facultad de Ingeniería y Ciencias, Matemáticas y Estadística, Universidad Adolfo Ibáñez, Chile):
Un modelo estructurado por edad-de-infección para estudiar la coinfección de VIH y VHS-2

17.20 **Elvis Gavilán** (CI^{2}MA & Departamento de Ingeniería Matemática, Universidad Universidad de Concepción, Chile):
Numerical solution of a spatio-temporal predator-prey model with infected prey

20.30 **Workshop Dinner**
Restaurante Torreón, Freire 1743, Concepción

[1]This event is supported by Conicyt projects PFB03 (CMM-Basal), CRHIAM CONICYT/Fondap/15130015, PCI/MEC/80170119, and Fondecyt 1170473 and 1181511.

Tuesday, June 19, 2018

09.00 **Fernando D. Córdova-Lepe** (Facultad de Ciencias Básicas, Universidad Católica del Maule, Talca, Chile):
An exposure model and development of infectious-contagious respiratory diseases

09.40 **Aníbal Coronel** (Departamento de Ciencias Básicas, Universidad del Bío-Bío, Chillán, Chile):
Some results for an inverse problem arising in a model of indirectly transmitted diseases

10.20 **Coffee break**

10.40 **Verónica Anaya** (Departamento de Matemática, Facultad de Ciencias, Universidad del Bío-Bío, Concepción):
A convergent finite volume scheme for an indirectly transmitted disease model

11.20 **Nolbert Morales** (Departamento de Matemática, Facultad de Ciencias, Universidad del Bío-Bío, Concepción):
An approximation to the minimum wave for Nicholson blowflies equation

12.00 **Helí Elorreaga Aldaz** (Departamento Departamento de Matemática, Facultad de Ciencias, Universidad del Bío-Bío, Concepción):
Dynamics of a Kolmogorov-type predator-prey model with two discrete delays

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Monday, June 18, 2018

15.00 **Gerardo Chowell** (School of Public Health, Georgia State University, Atlanta, GA, USA):

The scaling of epidemic growth in the spread of infectious diseases

The increasing use of mathematical models for epidemic forecasting has highlighted the importance of designing reliable models that capture the baseline transmission characteristics of specific pathogens and social contexts. Here, we review recent progress on modeling and characterizing early epidemic growth patterns from infectious disease outbreak data, and survey the types of mathematical formulations that are most useful for capturing a diversity of early epidemic growth profiles, ranging from sub-exponential to exponential growth dynamics. Specifically, we review mathematical models that incorporate spatial details or realistic population mixing structures, including meta-population models, individual-based network models, and simple SIR-type models that incorporate the effects of reactive behavior changes or inhomogeneous mixing. In this process, we also analyze simulation data stemming from detailed large-scale agent-based models previously designed and calibrated to study how realistic social networks and disease transmission characteristics shape early epidemic growth patterns, general transmission dynamics, and control of international disease emergencies such as the 2009 A/H1N1 influenza pandemic and the 2014-2015 Ebola epidemic in West Africa.

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15.40 **Tetsuro Kobayashi** (School of Public Health, Georgia State University, Atlanta, GA, USA):

Spatio-temporal and socio-demographic patterns of Chikungunya, Dengue, and Zika infections in Mexico in 2016

Chikungunya, Dengue, and Zika viral infections are vector-borne diseases that are endemic in Mexico. Here we analyze the relationship between epidemic size and climate data and socio-economic factors across the country. We collected weekly incidence data, daily climate data, and socio-economic status on each state of Mexico in 2016. The data sources are Mexican surveillance system, the Weather Underground, OECD.org, and INEGI Mexico. We measured the direct distances from the Oaxaca state (the southern most state) to all other states and compared them with the timing of the state-level curves. Chikungunya and Dengue both show "south-to-north" spreading patterns especially in the states that are located along the coast lines of Mexico. Chikungunya and Zika infections are especially prevalent among the moderately-populated cities, while all three infections are prevalent in the largest-sized cities. The coastline and south-to-north patterns of spreading may be a good predictor of when a seasonal outbreak starts in each state, which could guide public health interventions.

16.20 **Coffee break**

16.40 **Katia Vogt** (Facultad de Ingeniería y Ciencias, Matemáticas y Estadística, Universidad Adolfo Ibáñez, Chile):

Un modelo estructurado por edad-de-infección para estudiar la coinfección de VIH y VHS-2

Existe evidencia de una correlación entre la prevalencia de VHS-2– una infección viral incurable que se caracteriza por reactivación periódica del virus– y la prevalencia de VIH en la población humana. Preseántaremos un modelo matemático determinista de ecuaciones diferenciales, que modela la dinámica de la coinfecciöún de VHS-2 y VIH. Incorporamos una variable que representa edad-de-infección de VHS-2 para rastrear los perodos alternantes de infectividad de dicha enfermedad. El modelo considera relaciones heterosexuales y diferencia tres grupos poblacionales: hombres, mujeres comunes y mujeres trabajadoras sexuales. Mostraremos una expresión para el nmero reproductivo básico y el nmero reproductivo de invasión, que determinan la capacidad de invasión de una enfermedad sin y con la presencia de la otra, respectivamente. Evaluaremos el efecto de la coinfección de VIH y VHS-2 sobre la prevalencia de VIH, y también discutiremos el rol de los tres grupos poblacionales en la propagación del VIH.

17.20 **Elvis Gavilán** (CI^{2}MA & Departamento de Ingeniería Matemática, Unversidad de Concepción):

Numerical solution of a spatio-temporal predator-prey model with infected prey

A spatio-temporal eco-epidemiological model is formulated by combining an available non-spatial model for predator-prey dynamics with infected prey [D. Greenhalgh and M. Haque, *Math. Meth. Appl. Sci.*, **30** (2007), 911–929] with a spatio-temporal susceptible-infective (SI)-type epidemic model of pattern formation due to diffusion [G.-Q. Sun, *Nonlinear Dynamics*, **69** (2012), 1097–1104]. It is assumed that predators exclusively eat infected prey, in agreement with the hypothesis that the infection weakens the prey and increases its susceptibility to predation. Furthermore, the movement of predators is described by a non-local convolution of the density of infected prey as proposed in [R.M. Colombo and E. Rossi, *Commun. Math. Sci.*, **13** (2015), 369–400]. The resulting convection-diffusion-reaction system of three partial differential equations for the densities of susceptible and infected prey and predators is solved by an efficient method that combines weighted essentially non-oscillatory (WENO) reconstructions and an implicit-explicit Runge-Kutta (IMEX-RK) method for time stepping. Numerical examples illustrate the formation of spatial patterns involving all three species. Future directions of research are suggested. This presentation is based on joint work with R. Bürger, G. Chowell, P. Mulet, and L.M. Villada.

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09.00 **Fernando D. Córdova-Lepe** (Facultad de Ciencias Básicas, Universidad Católica del Maule, Talca, Chile):

An exposure model and development of infectious-contagious respiratory diseases

An epidemiological mathematical model that represents the interaction between intoxications due to pesticides and infectious-contagious respiratory diseases, with variable of interest, the application of prevention treatments for agricultural workers exposed constantly to this type of toxic, which is presented. This model is represented by a system of ordinary differential equations, which is epidemiological analyzed by obtaining and based on a reproductive number theory, complemented through numerical simulations. This presentation is based on joint work with J.P. Gutiérrez-Jara and M.T. Muñoz-Quezada.

09.40 **Aníbal Coronel** (Departamento de Ciencias Básicas, Universidad del Bío-Bío, Chillán, Chile):

Some results for an inverse problem arising in a model of indirectly transmitted diseases

This contribution deals with the problem for the well posedness for the coefficients identification on diffusion-reaction system modelling the indirectly transmitted diseases. Let us consider that Ω_1 and Ω_2 are two open subsets of $\mathbb{R}^n$, $n = 1, 2, 3$, then the direct problem is given by the following nonlinear system

$$\begin{aligned}
\partial_t\varphi - \operatorname{div}(d_{11}(x)\nabla\varphi) &= F_1(x,\varphi,\psi,\chi,c), && \text{in } Q_{1,T} =]0,T[\times\Omega_1,\\
\partial_t\psi - \operatorname{div}(d_{12}(x)\nabla\psi) &= F_2(x,\varphi,\psi,\chi,c), && \text{in } Q_{1,T},\\
\partial_t\chi - \operatorname{div}(d_{13}(x)\nabla\chi) &= F_3(x,\varphi,\psi,\chi,c), && \text{in } Q_{1,T},\\
\partial_t u - \operatorname{div}(d_{11}(x)\nabla u) &= G_1(x,u,v,w,c), && \text{in } Q_{2,T} =]0,T[\times\Omega_2,\\
\partial_t v - \operatorname{div}(d_{12}(x)\nabla v) &= G_2(x,u,v,w,c), && \text{in } Q_{2,T},\\
\partial_t w - \operatorname{div}(d_{13}(x)\nabla w) &= G_3(x,u,v,w,c), && \text{in } Q_{2,T},\\
\partial_t c - K(x,u,v,w,\varphi,\psi,\chi,c) &= 0, && \text{in } Q_T = \Omega_1\cup\Omega_2\times]0,T[,\\
d_{11}(x)\frac{\partial\varphi}{\partial\eta_1} = d_{12}(x)\frac{\partial\psi}{\partial\eta_1} &= d_{13}(x)\frac{\partial\chi}{\partial\eta_1} = 0, && \text{on }]0,T[\times\partial\Omega_1,\\
d_{21}(x)\frac{\partial u}{\partial\eta_2} = d_{22}(x)\frac{\partial v}{\partial\eta_2} &= d_{23}(x)\frac{\partial w}{\partial\eta_2} = 0, && \text{on }]0,T[\times\partial\Omega_2,\\
(\varphi,\psi,\chi)(x,0) &= (\varphi_0,\psi_0,\chi_0,u_0)(x) && \text{in } \Omega_1,\\
(u,v,w)(x,0) &= (u_0,v_0,w_0)(x) && \text{in } \Omega_2,\\
c(x,0) &= c_0(x) && \text{in } \Omega_1\cup\Omega_2,
\end{aligned}$$

with

$$
\begin{aligned}
F_1 &= -\sigma_{11}(x)\frac{\varphi\psi}{H_1} - \sigma_{31}(x)c\varphi + (1-w_1)\lambda_1\psi + b(x)H_1 - (m(x)+k(x)H_1)\varphi,\\
F_2 &= \sigma_{11}(x)\frac{\varphi\psi}{H_1} + \sigma_{31}(x)c\varphi - \lambda_1\psi - (m(x)+k(x)H_1)\psi,\\
F_3 &= w_1\lambda_1\psi - (m(x)+k(x)H_1)\chi,\\
G_1 &= -\sigma_{32}(x)cu, \qquad G_2 = \sigma_{32}(x)cu - \lambda_2 v, \qquad G_3 = \varepsilon_1\lambda_2 v,\\
K &= \sigma_{13}(x)(1-c)\tilde{\psi} + \sigma_{23}(x)(1-c)\tilde{v} - \delta(x)c,
\end{aligned}
$$

where (φ, ψ, χ) represent population densities of the subclasses of susceptible, infective and recovered individuals from the total population $H_1 = \varphi + \psi + \chi$; (u, v, w) represent population densities of the susceptible, infective and recovered subclasses of the total population $H_2 = u+v+w$; the variable c represents the proportion of contaminated habitat or environment; b represents the birth-rate which is identical in each subclass; $m(x) + k(x)H_1$ represents a spatially variable carrying capacity; $1/\lambda_i$ represents the duration of the infective stage in population $H_i, i = 1, 2$; $\tilde{\psi}$ and $\tilde{v}$, denotes the prolongation by zero of ψ and v, on Ω_1 and Ω_2, respectively; and $d_{ij} : \mathbb{R}^n \to \mathbb{R}$ are continuous functions such that

$$
\begin{aligned}
&\exists M_{ij}, C > 0 \quad : \quad M_{ij} \le d_{ij} \quad \text{and}\\
&\quad |d_{ij}(I_1) - d_{ij}(I_2)| \le C|I_1 - I_2| \quad \text{for all} \quad I_1, I_2 \in \mathbb{R}^n.
\end{aligned}
$$

Given some measurement of all variables are given at final time T on their respective spatial domains of definition, the inverse problem is formulated as follows: “Find the coefficients of the model σ_{11}, σ_{13}, σ_{23}, σ_{31}, σ_{32}, b, m, k, δ, such that at time $t = T$ the solution of the direct problem is as close as to the observed data.” The inverse problem is formalized by the introduction of an appropriate objective function, then we apply the optimization with partial differential constraints theory, to deduce the necessary optimality conditions, the stability and the uniqueness of the inverse problem.

10.20 Coffee break

10.40 **Verónica Anaya** (Departamento de Matemática, Facultad de Ciencias, Universidad del Bío-Bío, Concepción):

A convergent finite volume scheme for an indirectly transmitted disease model

This work is concerned with a model of the indirect transmission of an epidemic disease between two spatially distributed host populations having non-coincident spatial domains with nonlocal and cross-diffusion, the epidemic disease transmission occurring through a contaminated environment. The mobility of each class is assumed to be influenced by the gradient of the other classes. We address the questions of existence of weak solutions by using a regularization method. Moreover, we propose a finite volume scheme and proved the well-posedness, nonnegativity and convergence of the discrete solution. The convergence proof is based on deriving a series of a priori estimates and by using a general L^p compactness criterion. Finally, the numerical scheme is illustrated by some examples.

11.20 **Nolbert Morales** (Departamento de Matemática, Facultad de Ciencias, Universidad del Bío-Bío, Concepción):

An approximation to the minimum wave for Nicholson blowflies equation

In this work [1], we will present the approximation of traveling waves solution propagated at minumum speeds $c_0(h)$ (critical case) of the delayed Nicholson blowflies equation

$$\begin{aligned} & u_t(t,x) = \Delta u(t,x) - \delta u(t,x) + pu(t-\hat{h},x)e^{-u(t-\hat{h},x)}, \\ & u(t,x) \geq 0, \quad x \in \mathbb{R}^m, \end{aligned} \tag{1}$$

where $\hat{h} \geq 0$ and the parameters p, δ satisfy $p/\delta \in (1, e]$. In order to do that, we construct a super and sub solution to (1). Also, by that construction, an alternative proof of existence of traveling waves moving at minimum speed is given. The main difficulty in this case is due by the multiplicity of the eigenvalue associated with the linearization about 0 equilibrium, where an adequate, and different to the super-critical case, sub-solution is required. Our main theorem is

Theorem 1 *Let $p/\delta \in (1,e]$, $h \geq 0$ and $c = c_0(h)$. Then, Equation (1) possesses a traveling wave solution $u(t,x) = \phi(\nu \cdot x + ct)$. Moreover, its profile can be obtained as $\phi(t) = \lim_{n\to\infty} \phi_n(t)$, for all $t \in \mathbb{R}$, with defined by induction as follows:*

$$\phi_0(t) = \overline{\phi}(t) := \begin{cases} -k_1(t-t_0)e^{\lambda_1(t-t_0)}, & \text{if } t < 0, \\ \kappa - k_2 e^{(\mu_1-\epsilon_1)t}, & \text{if } t \geq 0, \end{cases} \tag{2}$$

with $t_0, \epsilon_1, k_1, k_2$ defined by

$$t_0 := \frac{2}{\lambda_1}, \quad \epsilon_1 := \frac{-c_0 + 2\mu_1 + \sqrt{(c_0 - 2\mu_1)^2 + 4\ln(e\delta/p)e^{-r\mu_1}}}{2},$$
$$k_1 := \frac{\kappa\lambda_1(\epsilon_1 - \mu_1)e^2}{\lambda_1 + 2(\epsilon_1 - \mu_1)}, \quad k_2 := \frac{\kappa\lambda_1}{\lambda_1 + 2(\epsilon_1 - \mu_1)}$$

and

$$\phi_{n+1}(t) := \frac{p}{\delta(\alpha_2 - \alpha_1)} \int_{-\infty}^{t} e^{\alpha_1(t-s)}\phi_n(s-r)e^{-\phi_n(s-r)}ds + \frac{p}{\delta(\alpha_2 - \alpha_1)} \int_{t}^{\infty} e^{\alpha_2(t-s)}\phi_n(s-r)e^{-\phi_n(s-r)}ds$$

for all $n \geq 0$.

This research was supported in part by the FONDECYT grants 11130367 (A. Gómez).

References

[1] Gomez A, Morales N. An approximation to the minimum traveling wave for the delayed diffusive Nicholson's blowflies equation, DOI: 10.1002/mma.4401

[2] Wu J, Zou X. Erratum to "Traveling wave fronts of reaction-diffusion systems with delays"[J. Dynam. Diff. Eq. 13,651,687(2001)], J. Dynam. Diff. Eq. 20 (2008) 531–532.

12.00 **Helí Elorreaga Aldaz** (Departamento Departamento de Matemática, Facultad de Ciencias, Universidad del Bío-Bío, Concepción):

Dynamics of a Kolmogorov-type predator-prey model with two discrete delays

In this work we consider a Kolmogorov-type predator-prey model with two discrete delays:

$$\begin{aligned} \dot{x}(t) &= x(t)f(x(t-\tau_1), y(t)), \\ \dot{y}(t) &= y(t)g(x(t), y(t-\tau_2)) \end{aligned} \tag{3}$$

Firstly, we study the absolute stability and conditional stability of the system by analyzing its associated characteristic equation. By choosing the delay as the bifurcation parameter, we show that Hopf bifurcation can occur as the delay passes through some critical values. Using the normal form theory and central manifold argument, we establish the direction and stability of Hopf bifurcation. Finally, we present an example with numerical simulations in order to verify the theoretical results obtained.

References

[1] B.D. Hassard, N.D. Kazarinoff and Y.H. Wan, Theory and Applications of Hopf Bifurcation, Cambridge University Press, Cambridge, UK, 1981.

[2] S. Ruan, Absolute stability, conditional stability and bifurcation in Kolmogorov - type predator - prey systems with discrete delays, *Quarterly of applied mathematics.*, **59** (2001), 159–173.

[3] S. Ruan and J. wei, On the zeros of transcendental functions with applications to stability of delay differential equations with two delays, *Mathematical Analysis.*, **10** (2003), 863–874.

[4] H.L. Smith, An Introduction to Delay Differential Equations with Applications to the Life Sciences, Springer. New York.